Min Max

Q1: SOP / POS

1) Simplify using bool alg / K-map:

$F(A,B,C) = \Sigma(0,1,4,6)$

$F(A,B,C) = \Sigma(m_0, m_1, m_4, m_6)$

$f = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$

000 100 001 110

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1 (000)	1 (001)	0 (011)	0 (010)
A 1	1 (100)	0 (101)	0 (111)	1 (110)

$f = \bar{A}\bar{B} + A\bar{C} + \bar{B}\bar{C}$ ← No A as 0

$f = \bar{A}\bar{B} + A\bar{C}$

SOP

A \ B	$\bar{B}$ 0	B 1
$\bar{A}$ → 0	0 (00)	0 (01)
A → 1	1 (10)	1 (11)

$f = A\bar{B} + AB$

SOP

$F(A,B,C,D) = \Sigma(0,2,3,6,7,8,10,15)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	1	0	1	1
$\bar{A}B$ 01	0	0	1	1
AB 11	0	0	1	0
$A\bar{B}$ 10	1	0	0	1

$f = \bar{A}C + BCD + \bar{B}\bar{D}$

SOP

POS

$Q = \Pi(0,1,4,8,9,12,13)$

AB \ CD	$\bar{C}\bar{D}$ 00	$\bar{C}D$ 01	CD 11	$C\bar{D}$ 10
$\bar{A}\bar{B}$ 00	0	0	1	1
$\bar{A}B$ 01	0	1	1	1
AB 11	0	1	0	1
$A\bar{B}$ 10	0	0	1	1

$\bar{f} = \bar{C}\bar{D} + \bar{A}\bar{C} + ABCD$

$f = \overline{\bar{C}\bar{D} + \bar{A}\bar{C} + ABCD}$

$f = (\overline{\bar{C}\bar{D}})(\overline{\bar{B}\bar{C}})(\overline{ABCD})$

$f = (\bar{\bar{C}} + \bar{\bar{D}})(\bar{\bar{B}} + \bar{\bar{C}})(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

$f = (C+D)(B+C)(\bar{A} + \bar{B} + \bar{C} + \bar{D})$

Bool algebra:

$x \cdot 0 = 0$ $\quad x + 0 = x$

$x \cdot 1 = x$ $\quad x + 1 = 1$

$x \cdot x = x$ $\quad x + x = x$

$x \cdot \bar{x} = 0$ $\quad x + \bar{x} = 1$

$\bar{\bar{x}} = x$

Distributive eq

$x \cdot (y + z) = x \cdot y + x \cdot z \iff x + y \cdot z = (x+y) \cdot (x+z)$

$x + x \cdot y = x \iff x \cdot (x+y) = x$ → Absorption

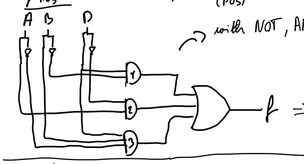
$x \cdot y + x \cdot \bar{y} = x \iff (x+y) \cdot (x + \bar{y}) = x$ → combining

$\overline{x \cdot y} = \bar{x} + \bar{y} \iff \overline{x + y} = \bar{x} \cdot \bar{y}$ → DeMorgan's theorem

$x + \bar{x} \cdot y = x + y \iff x \cdot (\bar{x} + y) = x \cdot y$ → Consensus

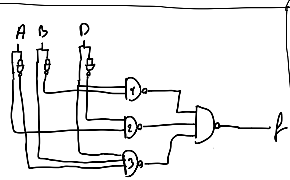
2) Function → NAND (SOP) / NOR (POS) gates

f POS:

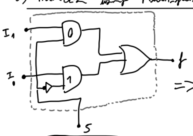


with NOT, AND, OR

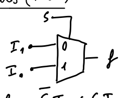
⇒ NAND only ⇒



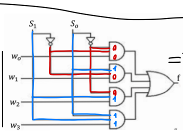
3) Function using Multiplexers (MUX):



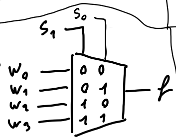
⇒



$f = \bar{S}I_0 + SI_1$



⇒



$f = w_0\bar{S}_1\bar{S}_0 + w_1\bar{S}_1S_0 + w_2S_1\bar{S}_0 + w_3S_1S_0$

Mux: 2 to 1:

$f = w_1w_2 + w_1w_3 + w_2w_3$

↳ $f = \bar{w}_1 f(0\ldots) + w_1 f(1\ldots)$

$f = \bar{w}_1(0 \cdot w_2 + 0 \cdot w_3 + w_2w_3) + w_1(1 \cdot w_2 + 1 \cdot w_3 + w_2w_3)$

$f = \bar{w}_1(w_2w_3) + w_1(w_2 + w_3 + w_2w_3)$

$f = \bar{w}_1(w_2w_3) + w_1(w_2 + w_3)$ ↳ $x + x \cdot y = x$ $\quad (x+y) + x \cdot y = x + y$

$f = \bar{w}_1 g + w_1 h$ → expand by w_2

$g = w_2w_3$

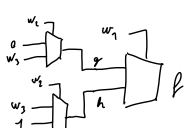
$g = \bar{w}_2(0 \cdot w_3) + w_2(1 \cdot w_3)$

$= \bar{w}_2(0) + w_2(w_3)$

$h = w_2 + w_3$

$h = \bar{w}_2(0 + w_3) + w_2(1 + w_3)$

$h = \bar{w}_2(w_3) + w_2(1)$



4 to 1:

$f = w_1w_2 + w_1w_3 + w_2w_3$

$f = \bar{w}_1\bar{w}_2(0 + 0 + 0) +$

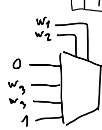
$\bar{w}_1w_2(0 + 0 + w_3) +$

$w_1\bar{w}_2(0 + w_3 + 0) +$

$w_1w_2(1 + w_3 + w_3)$

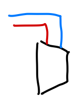
$= \bar{w}_1\bar{w}_2(0) + \bar{w}_1w_2(w_3) +$

$w_1\bar{w}_2(w_3) + w_1w_2(1)$



2 to 1 mux → 1 Variable

4 to 1 mux → 2 Variables



- most significant
- least significant

4) DeMorgan's theorem:

$\overline{x \cdot y} = \bar{x} + \bar{y}$

$\overline{x + y} = \bar{x} \cdot \bar{y}$

$\underset{\text{K-map}}{\overline{SOP}} = \underset{\text{K-map}}{POS}$

POS: $(\ldots + \ldots) \cdot (\ldots + \ldots)$

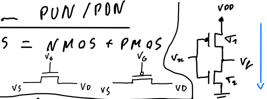
SOP: $(\ldots \cdot \ldots) + (\ldots \cdot \ldots)$

$\bar{w}_n \to 0$

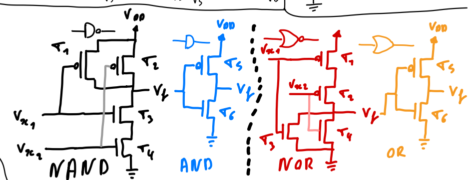
$w_n \to 1$

Q2: Function PUN / PDN

1) CMOS = NMOS + PMOS



x	T_1	T_2	f
0	on	off	1
1	off	on	0



x_1	x_2	T_1	T_2	T_3	T_4	NAND	T_5	T_6	AND	T_1	T_2	T_3	T_4	NOR	T_5	T_6	OR
0	0	on	on	off	off	1	off	on	0	on	on	off	off	1	off	on	0
0	1	on	off	on	off	1	off	on	0	on	off	on	off	0	on	off	1
1	0	off	on	off	on	1	off	on	0	off	on	off	on	0	on	off	1
1	1	off	off	on	on	0	on	off	1	off	off	on	on	0	on	off	1

f → CMOS: + → parallels, • → series $\qquad \bar{y}_N = y_P$

$y = \overline{A\bar{B} + \bar{B}CD} = \overline{\bar{B}(A + CD)}$

$y_N = \bar{B}(A + CD)$ → Bottom

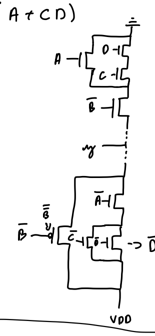
$y_P = \overline{\bar{B}(A + CD)}$

Top $= \bar{\bar{B}} + \overline{A + CD}$

$= \bar{\bar{B}} + \bar{A} \cdot \overline{CD}$

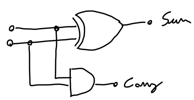
$= \bar{\bar{B}} + \bar{A} \cdot (\bar{C} + \bar{D})$

opposite B, opposite A, opposite C, opposite D



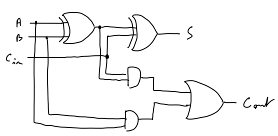
Half Adder:

x	y	Sum	Carry
0	0	0	0
1	0	1	0
0	1	1	0
1	1	0	1

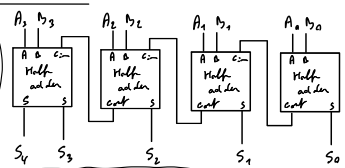


Full adder:

C_{in}	I_1	I_2	C_o	S
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1



4 Bit adder:



Overflow: Sign bit is wrong for the equation

- if the result is $> 2^{N-1} - 1$
- if the result is $< -2^{N-1}$

(N is # bits)

8 Bit

Unsigned

0 → 255

7 6 5 4 3 2 1 0

1 0 0 0 1 0 1 1

$2^7 + 2^3 + 2^1 + 2^0$

$= 139_{10}$

Signed

-128 → 127

7 6 5 4 3 2 1 0

+ 0 0 0 1 0 1 0 0

$2^4 + 2^2 = 20_{10}$

7 6 5 4 3 2 1 0

- 1 1 0 1 0 1 1 0

$-2^7 + 2^6 + 2^4 + 2^2 + 2^1$

$= -42_{10}$